Int. J. Solids Structures, 1965, Vol. 1, pp. 371 to 375. Pergamon Press Ltd. Printed in Great Britain

CONTINUUM THEORY OF DISLOCATION LOOPS

TAI TE WU

Division of Engineering, Brown University, Providence, Rhode Island

Abstract—For dislocation loops, the zeroth moment of dislocation strength always vanishes so that the first moment must be defined in order to give a continuum description. A second order tensor is thus introduced to account for the dislocation moment. This physical quantity contains more information than the macroscopic plastic strain. Its connection with the mathematical theory of slip is illustrated.

1. INTRODUCTION

THE continuum theory of dislocations has received much attention recently, and a review of this theory can be found in de Wit [1] and Bilby's [2] articles. Kroupa [3], Kröner [4] and Mura [5] have studied dislocation loops. It is the purpose of the present paper to give a more detailed account of these loops so that relations between the dislocation network and the gross plastic strain may be established without enumerating the number of loops.

2. REVIEW OF PREVIOUS WORK

Around any dislocation line, draw an arbitrary circuit by connecting lattice points. Also, pick any point in a perfect crystal and draw a similar circuit. In Fig. 1, an edge dislocation is shown, and the circuits are indicated by $(1, 2, \ldots, 8, 1')$ and $(1, 2, \ldots, 8, 1)$ respectively. The vector joining 1 and 1′ is known as the Burger's vector b_i. Let the physical scales be roughly indicated as follows:

(a) interatomic distance ~ 10 Å,
(b) interdislocation distance $\sim 10^3$ Å,
(c) macroscopic 'point' $\sim 10^5$ Å, and
(d) specimen size $\sim 10^8$ Å.

Then, the circuit has a size between (a) and (b).

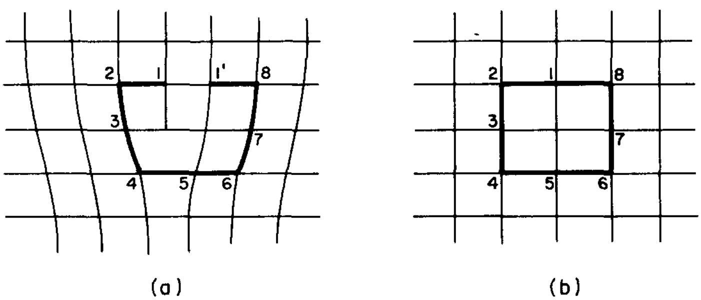

FIG. 1.

The first attempt in the continuum theory of dislocations is to consider a scale between (b) and (c). Draw a similar but much larger circuit than these shown in Fig. 1, which may be broken into many small ones encircling individual dislocations. If we designate the Burger's vector associated with the large circuit as B_i, then

$$B_i = \sum_n b_i^{(n)}, \tag{1}$$

where n indicates the nth dislocation. We can go to the continuum approximation by conjecturing that

$$B_i = \int_C \mathrm{d}u_i, \tag{2}$$

where C indicates the large circuit, and $\mathrm{d}u_i$ is the differential displacement. If B_i is not identically zero, u_i is discontinuous, but note that this is not the usual macroscopic displacement. Certain formal manipulations have been performed by defining a continuous lattice deformation in the configurational space [1], [2]. Let the specimen be cut by a plane denoted by S_i and draw a close circuit C in this plane. The quantity defined by equation (2) is presumably measurable physically, so that the tensor α_{ij} is defined by

$$B_i = \int \alpha_{ij}\, \mathrm{d}S_j. \tag{3}$$

In physical reality, as shown by X-ray micrographs [6], [7], the dislocations are generally in the form of loops. Therefore, in equation (1), for each $b_i^{(n)}$, there is a $b_i^{(m)}$ which cancels $b_i^{(n)}$; or in general, as long as there is no loose end present in the dislocation network, $B_i = 0$. Thus, in the subsequent analysis, we shall take $B_i \equiv 0$, so that u_i is single valued in the macroscopic sense where a point has a size of scale (c).

3. DESCRIPTION OF THE LOOPS

Since $B_i \equiv 0$, in the continuum theory, it is required to define another quantity in order to give some information about the dislocation network. Define

$$H_{ji} = \frac{1}{A}\int_C x_j\, \mathrm{d}u_i, \tag{4}$$

where A is the area enclosed by C, and x_j the coordinate vector. Since u_i is single valued, $\mathrm{d}u_i = u_{i,j}\,\mathrm{d}x_j$, so that equation (4) gives

$$\begin{aligned} H_{ji} &= \frac{1}{A}\int_C x_j u_{i,k}\, \mathrm{d}x_k \\ &= \frac{1}{A}\int \epsilon_{lhk}(x_j u_{i,k})_{,h}\, \mathrm{d}S_l \\ &= \frac{1}{A}\int \epsilon_{ljk} u_{i,k}\, \mathrm{d}S_l \end{aligned} \tag{5}$$

where ϵ_{ljk} is the permutation tensor.

Define a third rank tensor θ_{kji}, the dislocation moment density tensor, as

$$H_{ji} = \frac{1}{A}\int \theta_{kji}\, \mathrm{d}S_k, \tag{6}$$

so that

$$\theta_{kji} = \epsilon_{kjl} u_{i,l}. \tag{7}$$

Equation (7) can be inverted to give

$$u_{i,j} = \tfrac{1}{2}\epsilon_{klj}\theta_{kli}. \tag{8}$$

Physically, consider a piece of single crystal subjected to plastic deformation under simple tension and then unloaded. A dislocation network has thus been introduced, or more precisely, the virgin network has been altered. Then, according to equation (8), the plastic strain is defined as

$$\begin{aligned} \varepsilon_{ij}^{P} &= \tfrac{1}{2}(u_{i,j} + u_{j,i}) \\ &= \tfrac{1}{4}(\epsilon_{klj}\theta_{kli} + \epsilon_{kli}\theta_{klj}). \end{aligned} \tag{9}$$

It is therefore required to find θ_{kji} by direct physical measurements. In fact, this third order tensor contains some information of the loading history which is not specified by ε_{ij}^{P} measurements.

In equation (4), the definition of H_{ji} involves a contour C. Therefore, we must require that for a given dislocation network enclosed by C, H_{ji} should be contour independent as long as A is fixed. Consider an analogous situation in statics. Let a plate be subjected to a concentrated normal force F_i. Draw a contour K around it, and denote the differential force per unit length on the edge of the contour as $\mathrm{d}f_i$. Then,

$$\int_K \mathrm{d}f_i = F_i, \tag{10}$$

and

$$\int_K \epsilon_{ijk} x_k\, \mathrm{d}f_j = 0, \tag{11}$$

where x_k is measured from the point of application of F_i. The requirement of equation (11) is very similar to what is needed in the present continuum theory of dislocation loops. In equation (2), we may imagine the procedure of breaking C into many small contours each of which encircles one dislocation. From the contours in Fig. 1, we get

$$\int_{C_n} \mathrm{d}u_i = b_i^{(n)}, \tag{12}$$

where C_n is the contour around the nth dislocation. Equation (12) is similar to equation (10). Therefore, similar to equation (11), we shall demand that

$$\int_{C_n} x_j\, \mathrm{d}u_i = 0, \tag{13}$$

where x_j is measured from the center of the dislocation. Equation (13) indeed makes H_{ji} a physically meaningful quantity; however, it is only a conjecture that equation (13) should hold.

In Fig. 2, one dislocation loop is shown. By (13),

$$\begin{aligned}\int_{C_1} x_j\,du_i &= \int_{C_1} (X_j+y_j)\,du_i \\ &= X_j \int_{C_1} du_i.\end{aligned} \tag{14}$$

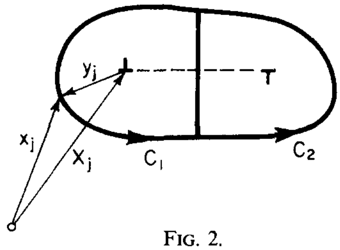

FIG. 2.

Therefore, in this case,

$$H_{ji} = \frac{1}{A} a_j b_i, \tag{15}$$

where a_j is the vector between the two dislocations. A fixed convention is required to define its direction, just as in the case of Burger's vector. In general

$$H_{ji} = \frac{1}{A} \sum_n a_j^{(n)} b_i^{(n)}. \tag{16}$$

4. AN EXAMPLE

From equations (6) and (16), we can construct the θ_{kji} for the slip theory. (Fig. 3.) The loop is made of two straight edge dislocations, and the Burger's vector is assumed to coincide with the direction of slip specified by $m_i^{(\tau)}$ for the τth system. Let l_i be the normal to the cutting plane, and $n_i^{(\tau)}$ be the normal to the τth slip plane. Then,

$$\theta_{kji} = \gamma_\tau \epsilon_{jlk} n_l^{(\tau)} m_i^{(\tau)}. \tag{17}$$

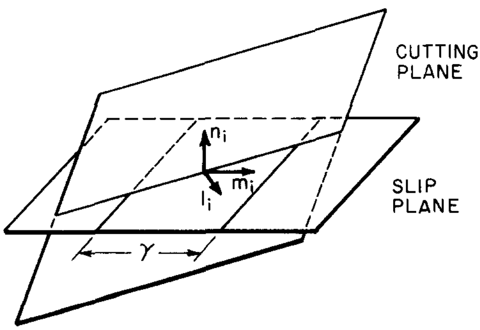

FIG. 3.

where γ_τ is the amount of slip of the τth system. Substituting equation (17) into equation (9) gives

$$\varepsilon^P_{ij} = \sum_\tau \gamma_\tau \tfrac{1}{2}(n_i^{(\tau)} m_j^{(\tau)} + m_i^{(\tau)} n_j^{(\tau)}), \tag{18}$$

which reproduced slip theory [8].

5. CONCLUSION

A precise meaning of the dislocation loop is given in the continuum theory, so that it is possible to relate dislocation movement to the macroscopic slip theory. The quantity H_{ji} is in principle measurable physically, so that for a given deformed crystal, a more refined quantity than the gross plastic strain is known. Plasticity rules may then be formulated in greater detail.

Acknowledgement—The author is greatly in debt to Professor B. Budiansky for suggesting the problem. This work was supported by ARPA SD-88 with Harvard University and SD-86 with Brown University.

REFERENCES

[1] R. de Wit, *Solid St. Phys.* **10**, 249 (1960).
[2] B. A. Bilby, *Prog. Solid Mech.* **1**, 329 (1960).
[3] F. Kroupa, *Czech. J. Phys.* **B12**, 191 (1962).
[4] E. Kröner, *J. Math. Phys.* **42**, 27 (1963).
[5] T. Mura, *Int. J. Engng. Sci.* **1**, 371 (1963).
[6] A. R. Lang, *J. appl. Phys.* **30**, 1748 (1959).
[7] J. J. Gilman and W. G. Johnston, *Solid St. Phys.* **13**, 147 (1962).
[8] S. B. Batdorf and B. Budiansky, *A Mathematical Theory of Plasticity Based on the Concept of Slip*, NACA TN 1871, 1949.

(*Received* 26 *November* 1964)

Résumé—Dans les boucles de dislocation le moment au zéro de la force de dislocation disparait toujours, de sorte que le moment d'origine doit être défini de manière à assurer la continuité. L'auteur introduit ainsi un tenseur du deuxième ordre faisant intervenir le moment de dislocation. Cette quantité physique donne plus d'informations que la déformation plastique macroscopique. L'auteur fait apparaître sa liaison avec la théorie mathématique du glissement.

Аннотация—Для петель смещения нулевой момент силы смещения всегда исчезает, так что необходимо определить первый момент, чтобы дать континуумное описание. Таким образом, внесен тензор второго порядка, чтобы учесть момент смещения. Эта физическая величина содержит больше информации, чем макроскопическая пластическая деформация. Показана ее связь с математической теорией сдвига.